Assignment 2

Textbook assignment: Chapter 2, "Conic Sections," pages 2-1 through 2-60 and Chapter 3, "Tangents, Normals, and Slopes of Curves," pages 3-1 through 3-33.

Learning Objective:

Recognize elements of conic sections and analyze equations of conic sections.

2-1. Conic sections are formed by passing planes at varying angles through a right circular cone.

1. True
2. False

2-2. All of the following are examples of conic sections except the

1. ellipse
2. circle
3. sphere
4. parabola

2-3. A conic section is the locus of all points in a plane whose distance from a fixed point is a constant ratio to its distance from a fixed line. This constant ratio is known as the

1. focus
2. directrix
3. coordinate
4. eccentricity

2-4. Which points satisfy the locus of the equation $x^2 + y^2 = 4$?

1. $(\pm 2,0),(0,\pm 2),(\pm\sqrt{2},\pm\sqrt{2})$
2. $(4,0),(0,4),(3,1)$
3. $(\pm 2,0),(0,\pm 2),(2,2)$
4. $(0,0)$

2-5. The distance from a point (x,y) to a point (3,1) is indicated by which of the following expressions?

1. $\sqrt{(x+3)^2 + (y+1)^2}$
2. $\sqrt{(x-3)^2 + (y-1)^2}$
3. $(x-1)^2 + (y-3)^2$
4. $(x+1)^2 + (y+3)^2$

2-6. If a point on a circle with its center at the origin has coordinates (4,3), the radius of the circle is

1. $\frac{4}{3}$
2. 3.5
3. 5
4. 7

2-7. The locus of the equation $(x - 3)^2 + (y - 5)^2 = 64$ is a circle. Where is its center located and what is its radius?

1. (5,3); 8 units
2. (3,5); 8 units
3. (3,5); 64 units
4. (5,3); 64 units

2-8. Three noncollinear points can determine the definition of

1. one circle
2. two circles of different diameters
3. three circles of different diameters
4. an infinite number of circles of different diameters

2-9. The equation of the circle passing through points (8,9), (-3,7), and (-4,-5) is

1. $x^2 + y^2 - 23x - 17y - 77 = 0$
2. $11x^2 + 11y^2 - 87x - 18y - 179 = 0$
3. $13x^2 + 13y^2 - 101x - 10y - 987 = 0$
4. $15x^2 + 15y^2 - 120x - 107y - 225 = 0$

2-10. The locus of all points equidistant from two given points is

1. a circle whose radius cannot be determined from the information given
2. a circle with a radius equal to the distance between the two given points
3. a circle with a radius equal to the square root of the distance between the two given points
4. the perpendicular bisector of the line segment connecting the two given points

2-11. A possible explanation for an inconsistent solution in attempting to determine the equation of a circle passing through three points is that the three points

1. are collinear
2. are noncollinear
3. form three equations with three unknowns
4. are not equidistant from each other

2-12. The parabola with its vertex at the origin in the Cartesian coordinate system, with its focus at (0,a) on the Y axis, and with the line y = -a as its directrix has the equation

1. $x^2 = -4ay$
2. $x^2 = 4ay$
3. $y^2 = -4ax$
4. $y^2 = 4ax$

2-13. The point that lies midway between the focus and directrix is known as the

1. vertex of the parabola
2. center of the parabola
3. origin of the parabola
4. midpoint of the parabola

2-14. The equation of the parabola $x^2 = 8y$ opens

1. to the right
2. to the left
3. downward
4. upward

2-15. The width of a parabolic curve at its focus is called the

1. vertex
2. center
3. directrix
4. focal chord

2-16. A parabola described by $y^2 = 16x$ has a horizontal distance between its focus and vertex of

1. 1
2. 4
3. 8
4. 16

2-17. $(y + 3)^2 = 16(x - 5)$ represents a parabola with its vertex at point

1. (-5,3)
2. (5,-3)
3. (-3,5)
4. (3,5)

2-18. Which of the following is true of an ellipse?

1. $e = 0$
2. $e = 1$
3. $e > 1$
4. $0 < e < 1$

2-19. The equation $\frac{x^2}{144} + \frac{y^2}{64} = 1$

describes an ellipse with a directrix parallel to the Y axis. Its semimajor and semiminor axes, respectively, are equal to

1. 6,4
2. 12,8
3. 18,8
4. 24,16

2-20. If an ellipse has its center at the origin, foci of (4,0) and (-4,0), and eccentricity of 0.8, what is the value of a in the formula $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$?

1. ±3.2
2. ±5
3. 25
4. ±25

2-21. If the center of an ellipse is moved from the origin to (-3,-2), the numerators of the fractions on the left side of the general equation for this ellipse are now

1. $(x + 3)^2$, $(y - 2)^2$
2. $(x - 3)^2$, $(y - 2)^2$
3. $(x + 3)^2$, $(y + 2)^2$
4. $(x - 3)^2$, $(y + 2)^2$

2-22. When completing the square of $3(A^2 - 18A)$ on the left side of an equation, the value to be added to the right side of the equation is

1. 36
2. 54
3. 81
4. 243

2-23. Reduce the equation

$$3x^2 + 4y^2 + 6x + 32y + 31 = 0$$

to the standard form of an ellipse. What are the lengths of the semimajor and semiminor axes, respectively?

1. $2\sqrt{3}$,3
2. $\sqrt{3}$,2
3. 1,4
4. 3,4

- The diagram in figure 2A refers to a hyperbola of form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

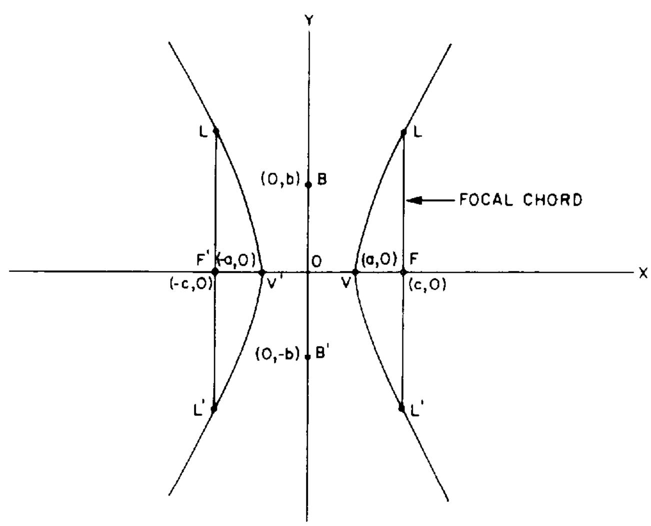

Figure 2A.--Hyperbola.

V and V' are the vertices of the hyperbola. V and V' may also be described as the intersections of the hyperbola with its focal axis. The focal axis, also called the principal axis, is the line segment FF' connecting the two foci of the hyperbola. That portion of the principal axis between V and V', that is, line segment VV', is called the transverse axis of the hyperbola and is equal to 2a.

Line segment BB', where BO = OB' = b, is called the conjugate axis of the hyperbola. Note that the conjugate axis is perpendicular to the transverse axis and its total length is equal to 2b.

The line segment LL' through either focus perpendicular to the principal axis is called the focal chord and is equal to $2b^2/a$.

2-24. The relationship between the foci and directrices of a hyperbola with respect to the origin as the center of symmetry is that

1. each focus is nearer the origin than its corresponding directrix
2. the focal axis is parallel to each directrix at the origin
3. each directrix is coincident with its corresponding focus at the origin
4. each focus is farther away from the origin than its corresponding directrix

2-25. The lines $y = -\frac{b}{a}x$ and $y = \frac{b}{a}x$ used in tracing a hyperbola are called

1. asymptotes
2. diagonals
3. center lines
4. directrices

2-26. $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ is described verbally with respect to the rectangular coordinate system as a hyperbola which opens

1. to the left
2. to the right
3. upward and downward
4. to the left and to the right

2-27. A hyperbola, with its center at the origin, has foci at (±5,0) and eccentricity of 1.5. In the standard form, this hyperbola would have a^2 equal to

1. 3 1/3
2. 10
3. 11 1/9
4. 13

2-28. The equation $\frac{x^2}{49} - \frac{y^2}{25} = 1$ is a hyperbola with its center at the origin and foci at

1. $\pm\sqrt{74}$ along the X axis
2. $\pm\sqrt{74}$ along the Y axis
3. ± 9 along the X axis
4. ± 9 along the Y axis

2-29. The focal chord of $\frac{x^2}{25} - \frac{y^2}{9} = 1$ is

1. 3 3/5
2. -21/5
3. 50/3
4. -16 2/3

2-30. What are the asymptotes of $\frac{x^2}{64} - \frac{y^2}{49} = 1$?

1. $y = \pm \frac{7}{8}x$
2. $8x \pm 7y = 0$
3. Both 1 and 2 above
4. $x = \pm \frac{7}{8}y$

2-31. $9x^2 + 4y^2 - 18x + 16y - 11 = 0$ is an equation of

1. a parabola
2. a hyperbola
3. an ellipse
4. a circle

Learning Objective:

Transform Cartesian coordinates to Polar coordinates and vice versa.

2-32. What must be known in order to locate a point in a plane by means of polar coordinates?

1. The horizontal and vertical components of the point
2. The radius vector and its polar angle
3. The radius vector and its chord
4. The polar angle and focal chord

2-33. The origin of a polar coordinate system is also called the

1. apex
2. vertex
3. center
4. pole

2-34. Polar angles are

1. positive if measured clockwise and negative if measured counterclockwise
2. negative if measured clockwise and positive if measured counterclockwise
3. always positive
4. positive for $0^\circ \leq \theta \leq 180^\circ$ and negative for $180^\circ < \theta < 360^\circ$

2-35. The equation $x^2 + y^2 = 2a$ expressed in polar coordinates is

1. $\rho^2 = 2a$
2. $\rho = 2a$
3. $\rho^2(\sin\theta + \cos\theta) = 2a$
4. $\rho(\sin^2\theta\ \cos^2\theta) = a$

2-36. Cartesian coordinates are the same as polar coordinates.

1. True
2. False

2-37. The equation $\rho = 4 \sin\theta \cot\theta$ is equivalent to

1. $4(x^2 + y^2) = x$
2. $x^2 + y^2 = 4x$
3. $\sqrt{x^2 + y^2} = 4x + \sqrt{x^2 + y^2}$
4. $\left(\frac{x^2}{\sqrt{x^2 + y^2}}\right)\left(\frac{y^2}{\sqrt{x^2 + y^2}}\right) = 4$

Learning Objective:

Calculate the slopes of curves and solve for the equation of the tangent line and normal line to a curve.

2-38. The slope of a curve at a point, P(x,y), can be represented by $\frac{\Delta x}{\Delta y}$.

1. True
2. False

2-39. Which of the following statements is TRUE regarding the error incurred in calculating the slope of a curve at a given point?

1. Calculation error is small if the curvature is great and large increments of Δy and Δx are used
2. Calculation error is large if the curve is nearly flat, regardless of the value of the Δx and Δy increments
3. If the curvature is great, calculation error increases as the lengths of Δy and Δx increase
4. If the curvature is great, calculation error decreases as the lengths of Δy and Δx increase

2-40. In finding the slope of a curve at a given point using sufficiently small increments, you are actually determining the slope of the tangent to the curve at that point.

1. True
2. False

2-41. A tangent to a curve at P(x,y) has a slope equal to 0. Relative to maximum and minimum, the curve at this point may be

1. at a minimum
2. at a maximum
3. at either a maximum or a minimum
4. at both a maximum and minimum

2-42. If the tangent to a curve at point P(x,y) has an infinite slope, this tangent is described as being perpendicular to

1. the curve at point P
2. the X axis
3. the Y axis
4. Δy

2-43. If $\Delta y = \frac{4\Delta x}{2y_1 + \Delta y}$ and y_1 is independent of both Δx and Δy, then as Δx approaches zero Δy approaches a value of

1. zero
2. $\frac{1}{2y_1}$
3. $\frac{4}{2y_1 + \Delta y}$
4. $\frac{4}{2y_1}$

2-44. If the slope of the tangent line to a curve at the point (5,10) is 3/2, the equation of the tangent line is

1. $2x - 3y + 10 = 0$
2. $3x - 2y + 5 = 0$
3. $5x - 10y + 12 = 0$
4. $10x - 5y + 24 = 0$

2-45. In using the general method to find the equation of the tangent to a curve at (x_1,y_1), pick a second point to the curve, $(x_1 + \Delta x, y_1 + \Delta y)$, and substitute these values into the equation of the curve and simplify. When simplifying, the term containing (Δy) to the second power can be eliminated because this term

1. is independent of Δx
2. is eliminated through subtraction of $(\Delta y)^2$
3. approaches zero as Δx approaches zero
4. is eliminated through division by $(\Delta y)^2$

2-46. If $x^2 + y^2 = r^2$ and (x_1,y_1) is a point on the curve, then the expression $r^2 - x_1^2 - y_1^2$ is equivalent to

1. zero
2. $r^2 - (x^2 - y^2)$
3. $r^2 + (x^2 - y^2)$
4. $r^2 - (-x^2 - y^2)$

2-47. The equation of the tangent to the curve $8x^2 - y^2 = 8$ at the point (3,8) is

1. $y = 3x + 5$
2. $y = -3x + 17$
3. $y = 3x - 17$
4. $y = 3x - 1$

2-48. The normal to a curve at a given point is

1. parallel to the tangent line at that point
2. perpendicular to the tangent line at that point
3. perpendicular to the X axis at that point
4. perpendicular to the Y axis at that point

2-49. The slope of a tangent to a curve at a given point is m_p. Therefore, the slope of the normal to the curve at this same point is the

1. inverse of m_p
2. negative of m_p
3. negative reciprocal of m_p
4. same as m_p

2-50. The length of the normal is defined as that portion of the normal line between the point P(x,y) and the

1. X axis
2. Y axis
3. origin
4. point P(x,0)

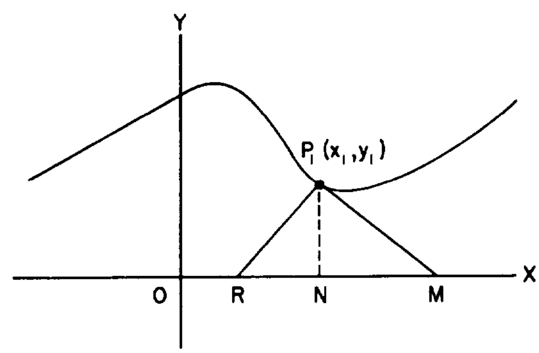

Figure 2B.--Curve with tangent and normal lines.

IN ANSWERING ITEM 2-51, REFER TO FIGURE 2B.

2-51. The lengths of the tangent and normal are represented, respectively, by

1. NM and RN
2. P_1M and P_1N
3. P_1N and P_1R
4. P_1M and P_1R

2-52. If $x^2 + y^2 = 4$, then the length of the tangent at $(1,\sqrt{3})$ is

1. $\sqrt{3}$
2. 2
3. 12
4. $\sqrt{12}$

Learning Objective:

Recognize parametric equations and determine coordinates of a tangent to a curve at a point when parametric equations are involved.

- In answering items 2-53 through 2-56, refer to the formulas $x = \pi r^2$ and $y = 2\pi r$.

2-53. The letter r is known as a

1. base
2. constant
3. parameter
4. parametric equation

2-54. If r is eliminated, the relationship between x and y is

1. $x = \frac{y^2}{2\pi}$
2. $x = \frac{y^2}{4\pi}$
3. $x = \frac{y}{2}$
4. $y^2 = \frac{x}{4\pi}$

2-55. If y = 2, what is the value of x?

1. π^2
2. 4π
3. $\frac{1}{\pi}$
4. $\frac{1}{4\pi^2}$

2-56. If $\frac{\Delta y}{\Delta r} = 2\pi$ and $\frac{\Delta x}{\Delta r} = 2\pi r$, the slope of the tangent to the curve at r = 3 is

1. $\frac{1}{\pi}$
2. $\frac{1}{3}$
3. 3
4. -3

- In answering items 2-57 and 2-58, refer to the following equations:

$x = 2 \sin \theta$, $y = 4 \cos \theta$

$\frac{\Delta x}{\Delta \theta} = 2 \cos \theta$, $\frac{\Delta y}{\Delta \theta} = -4 \sin \theta$

2-57. The general expression for the slope $\frac{\Delta y}{\Delta x}$ of the curve at (x_1,y_1) is

1. $-2 \tan \theta$
2. $-\frac{\tan \theta}{2}$
3. $-\frac{\cot \theta}{2}$
4. $2 \cot \theta$

2-58. At the point where $\theta = 30^\circ$ the equation of the normal to the curve is

1. $2x - y\sqrt{3} + 5 = 0$
2. $2y - x\sqrt{3} - 3\sqrt{3} = 0$
3. $6y - 3x - 9 = 0$
4. $6x - 3y + 8 = 0$

2-59. When using parametric equations, we find the vertical tangent to a curve by setting

1. $\frac{\Delta x}{\Delta t} = 0$
2. $\frac{\Delta y}{\Delta t} = 0$
3. $\frac{\Delta x}{\Delta y} = 0$
4. $\frac{\Delta y}{\Delta x} = 0$

2-60. If $x = t^2$, $y = 3t$, $\frac{\Delta x}{\Delta t} = 2t$, and $\frac{\Delta y}{\Delta t} = 3$, at what coordinates on the curve is the tangent parallel to the Y axis?

1. (4,6)
2. (1,1)
3. (0,1)
4. (0,0)